PROJECT OBJECTIVES

The research objectives within this project are the approach of the following research problems, which we hold hope to materialize in scientific research papers published on the 3-year duration of this project:

1. The description and classification of the types of bicrossed products of two given groups. This is a 70-year old still open problem. The first contributions in this direction of three of the members of the present grant were done this tear through a paper which was already submitted to publication. *The impact and relevance of the problem:* besides the interest for the problem itself as a group theoretic problem, a solution for this problem would have imediate consequences for the classification of finite dimensional Hopf algebras that factor through a group algebra and the dual of a group algebra for two groups that form a matched pair.

2. Description of the finite nonabelian simple groups that are not bicrossed products of two proper subgroups. *The impact and relevance of the problem:* The succes of such a result would provide a structure theorem for the problem of whether a finite nonabelian simple group is indecomposable.

3. We intend to introduce a new general product in algebra which would unify the two classical existing products: crossed products and bicrossed products (knit factorization). *The impact and relevance of the problem:* These completely different two types of products are present in many fields of mathematics: groups, Lie algebras, Lie groups, Hopf algebras, C*-algebras etc. The first steps made in this direction are quite promising. Applicability: we aim that this new type of very general product to open a new approach to the classification of finite dimensional Hopf algebras, and a new general cohomology theory. We suspect this would yield a fruitful theory that can hopefully lead to numerous published papers.

4. We intend to construct new classes of solutions for the fusion (pentagon) equation and then undergo a separate study of the fusion equation for sets. *Impact and applicability:* we will construct new types of finite quantum groups, obtained through the solutions of the fusion equation. At the same time, we hope to obtain a new FRT type theorem for the fusion equation for sets.

5. Generalization of the concept of integral - important in the Fourier transform theory - from the locally compact groups and Hopf algebras in a general algebraic setting of co-Frobenius coalgebras, through the consideration of the category of finite dimensional corepresentations, and the generalization of the results from Hopf algebras to co-Frobenius coalgebras. *Impact and applicatibility:* as an application of this line of research we aim to obtain a general duality theory for coalgebras that would generalize the Pntryagin-Tanaka-Krein dualities, that are present under one form or another, in algebra, analysis (different types of Fourier transform), topology, cryptoraphy (certain Fourier type transforms) etc.

6. Obtaining the complete characterization of coalgebras C with the property that the rational part of any module over the dual algebra C* is a direct summand in the module, a problem that falls under the general class of the so called splitting problems. *Impact and relevance:* the theory of coalgebras and their dual algebras.

7. In regards to category theory, corings and associated comodules, we aim to obtain a description of general categories such as categories of diagrams as equivalent to categories of comodules over some corings. *Impact, relevance and applicability:* This would generalize existing results for the category of (co)chain complexes of modules over a ring or results about the category of graded modules over a graded ring.

8. Possible big **applications in topology and homological algebra**: the different structures on the (co)homology of a topological space and the connections to Hopf algebras because of the existence of a multiplication and a comultiplication comultiplication bear the mark of the depth of very definition of these objetcs, that is, simplices and singular chains. The interpretation of this structure as a comultiplication associated to some objects that are more general than simplicial complexes, built by a certain topological model that is more general than the "simplex" model, can lead to the introduction of a generalized (co)homology theory in both topology and homological algebra.

9. Constructing of Hopf algebras with special properties: finite dimensional Hopf algebras, separable, co-Frobenius and quasi-triangular. This type of algebras can be constructed by using reconstruction FRT-type theorems which were used before in literature for this purpose. For example, it is possible to obtain finite dimensional Hopf algebras (Militaru), separable algebras (Canepeel, Ion, Militaru), quasi-triangular Hopf algebras (Drinfeld), but the possiblity of constructing Hopf algebras symultaneously having all these properties has not been investigated yet. *Impact and relevance:* Up to this point, there is still a lack of a certain structure within Hopf algebra theory, that would be relevant for applications and rigid enough to be classified (e.g. the analogue of simple groups in group theory). The sugested construction seems to fulfil these requirements.

10. Hopf algebras with triangular structure and Poincare-Birkhoff-Witt basesL the enveloping quantum algebras are examples of this type of Hopf algebras and a large part of the results concerning properties in group theory can be extended and we hope to achieve this in algebras. Moreover, algebras with these properties are algebraic deformations (this property is stronger than the one used in literature of formal deformation) of polynomial rings with triangular structure. *Impact and relevance:* enveloping quantum algebras are among the most structurally rich in the theory of Hopf algebras, having numerous connections to and applications in other domains. Through their properties, the objects we intend to study are very close of these enveloping quantum algebras, but with new relevance especially in algebraic geometry.

11. Topological constructions of Hecke algebras: we will produce such topological constructions for generalized double affine Hecke algebras introduced by Gan, Etingof and Oblomkov in 2007. New topological constructions that will emphasise the

automorphisms of these algebras as well as those of double affine Hecke algebras will be investigated. *Impact and relevance:* Double affince Hecke algebras are fundamental objects in modern mathematics; their oppening in topology paves the way for geometrical constructions of their representations, realized in this case as solution spaces for a special connection that acts on the algebra in question through monodromy.

12. Representation theory: this is an extended project, already in progress, and already with remarcable results (Bogdan Ion among the members of this project). We will continue to investigate MacDonald polynomials by a representation theoretic point of view. As it is already known, this involves several distinct branches of representation theory: for real groups,p-adic groups, Kac-Moody groups. Thus we will study the interacion between these types of groups (at the level of representaion theory) and we will pursue the implications, especially in regard to new formulas for multiplicities. *Impact and relevance:* The properties to be studied illustrate what Harish-Chandra calls the "Lefschetz" principle in representation theory. This principle states that certain results in representation theory hold independent of the basefield over which representations of groups are considered. 35 years after its formulation, the exact range of validity of this principle remains illusive. The proposed study contributes to clearing up this question in the case of finite dimensional spherical representations. Moreover, the geometric formulas for multiplicities are much more general than those obtained by Mirkovic and Vilonen (Annals of Math. 2006) and, at least until now, cannot be obtained by their methods.

13. A goal of this project is to begin and finalize the writing of a textbook for introduction to non-commutative algebra which would be of high standard as to be published by an international publishing house. It would addressed for MD and Ph.D studendts in mathematics, for the young mathematician about to be initiated in algebra and non-commutative algebra.